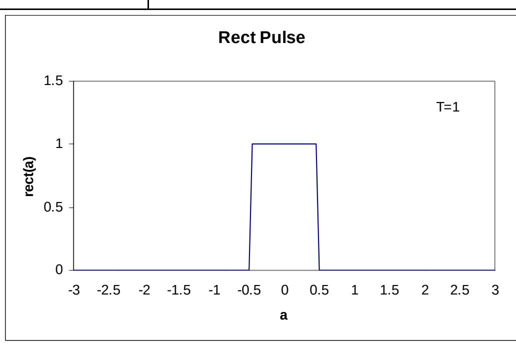
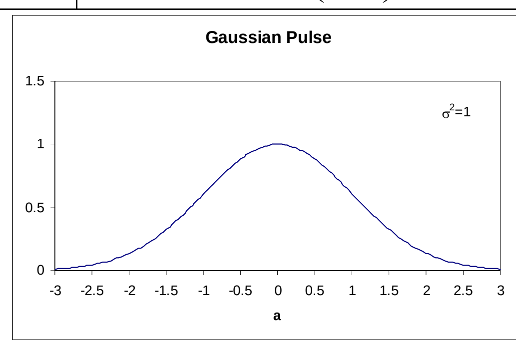
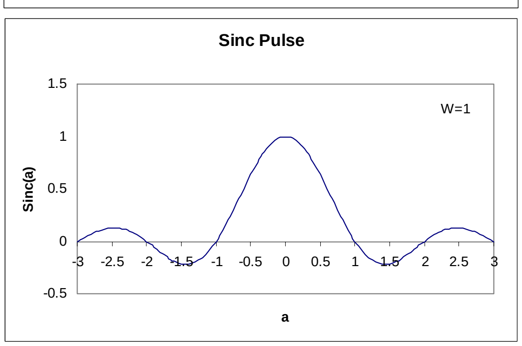
Table of Fourier Transform Pairs of Energy Signals

Function name	Time Domain x(t)	Frequency Domain X(ω)
FT	$x(t)$	$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt = \mathfrak{F}\{x(t)\}$
IFT	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega = \mathfrak{F}^{-1}\{X(\omega)\}$	$X(\omega)$
Rectangle Pulse	$rect\left(\frac{t}{T}\right) = \Pi\left(\frac{t}{T}\right) \equiv \begin{cases} 1 & \|t\| \le T/2 \\ 0 & elsewhere \end{cases}$	$T\,\mathrm{sinc}\left(\frac{T}{2\pi}\omega\right)$
Triangle Pulse	$\Lambda\left(\frac{t}{W}\right) \equiv \begin{cases} 1-\frac{2\|t\|}{W} & \|t\| \le W/2 \\ 0 & elsewhere \end{cases}$	$\frac{W}{2}\mathrm{sinc}^2\left(\frac{W}{4\pi}\omega\right)$
Sinc Pulse	$\mathrm{sinc}(Wt) \equiv \frac{\sin(\pi \cdot Wt)}{\pi \cdot Wt}$	$\frac{1}{W} rect\left(\frac{\omega}{2\pi W}\right)$
Exponen-tial Pulse	$e^{-a\|t\|} \quad a > 0$	$\frac{2a}{a^2+\omega^2}$
Gaussian Pulse	$\exp(-\frac{t^2}{2\sigma^2})$	$\left(\sigma\sqrt{2\pi}\right)\exp(-\frac{\sigma^2\omega^2}{2})$
Decaying Exponen-tial	$\exp(-at)u(t) \qquad \mathrm{Re}\{a\} > 0$	$\frac{1}{a+j\omega}$
Sinc^2 Pulse	$\mathrm{sinc}^2(Bt)$	$\frac{1}{B}\Lambda\left(\frac{\omega}{4\pi B}\right)$







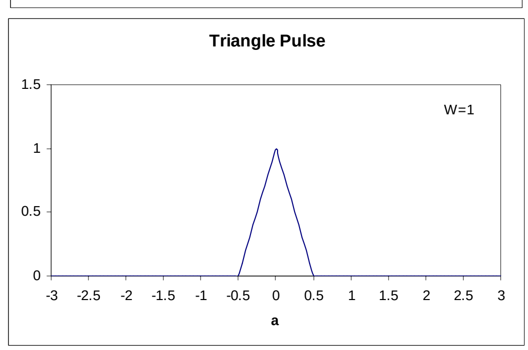


Table of Fourier Transform Pairs of Power Signals

Function name	Time Domain x(t)	Frequency Domain X(ω)
FT	$x(t)$	$X(\omega)=\int_{-\infty}^{\infty} x(t)\mathrm{e}^{-j\omega t}\,dt=\mathfrak{F}\{x(t)\}$
IFT	$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\mathrm{e}^{j\omega t}\,d\omega=\mathfrak{F}^{-1}\{X(\omega)\}$	$X(\omega)$
Impulse	$\delta(t)$	1
DC	1	$2\pi\delta(\omega)$
Cosine	$\cos(\omega_0 t+\theta)$	$\pi\left[e^{j\theta}\delta(\omega-\omega_0)+e^{-j\theta}\delta(\omega+\omega_0)\right]$
Sine	$\sin(\omega_0 t+\theta)$	$-j\pi\left[e^{j\theta}\delta(\omega-\omega_0)-e^{-j\theta}\delta(\omega+\omega_0)\right]$
Complex Exponential	$\exp(j\omega_0 t)$	$2\pi\delta(\omega-\omega_0)$
Unit step	$u(t)\equiv\begin{cases}1 & t\ge 0\\ 0 & t<0\end{cases}$	$\pi\delta(\omega)+\frac{1}{j\omega}$
Signum	$\mathrm{sgn}(t)\equiv\begin{cases}1 & t\ge 0\\ -1 & t<0\end{cases}$	$\frac{2}{j\omega}$
Linear Decay	$1/t$	$-j\pi\,\mathrm{sgn}(\omega)$
Impulse Train	$\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$	$\frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty}\delta\left(\omega-k\frac{2\pi}{T_s}\right)$
Fourier Series	$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, where $a_k=\frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$	$2\pi\sum_{k=-\infty}^{\infty} a_k\delta(\omega-k\omega_0)$

Table of Fourier Transforms of Operations

Operation	**FT Property** Given $g(t) \Leftrightarrow G(\omega)$
Linearity	$af(t) + bg(t) \Leftrightarrow aF(\omega) + bG(\omega)$
Time Shifting	$g(t - t_0) \Leftrightarrow e^{-j\omega t_0}\, G(\omega)$
Time Scaling	$g(at) \Leftrightarrow \frac{1}{\lvert a \rvert} G\left(\frac{\omega}{a}\right)$
Modulation (1)	$g(t)\cos(\omega_0 t) \Leftrightarrow \frac{1}{2}\left[G(\omega - \omega_0) + G(\omega + \omega_0)\right]$
Modulation (2)	$g(t)e^{j\omega_0 t} \Leftrightarrow G(\omega - \omega_0)$
Differentiation	If $f(t) = \frac{d\,g(t)}{dt}$, then $F(\omega) = j\omega \cdot G(\omega)$
Integration	If $f(t) = \int_{-\infty}^{t} g(\alpha)d\alpha$, then $F(\omega) = \frac{1}{j\omega} G(\omega) + \pi G(0)\delta(\omega)$
Convolution	$g(t) * f(t) \Leftrightarrow G(\omega) \cdot F(\omega)$, where $g(t) * f(t) \equiv \int_{-\infty}^{\infty} g(\alpha) f(t - \alpha)\, d\alpha$
Multiplication	$f(t) \cdot g(t) \Leftrightarrow \frac{1}{2\pi} F(\omega) * G(\omega)$
Duality	If $g(t) \Leftrightarrow z(\omega)$, then $z(t) \Leftrightarrow 2\pi g(-\omega)$
Hermitian Symmetry	If g(t) is real valued then $G(-\omega) = G^*(\omega)$ ($\lvert G(-\omega) \rvert = \lvert G(\omega) \rvert$ and $\angle G(-\omega) = -\angle G(\omega)$)
Conjugation	$g^*(t) \Leftrightarrow G^*(-\omega)$
Parseval's Theorem	$P_{avg} = \int_{-\infty}^{\infty} \lvert g(t) \rvert^2\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert G(\omega) \rvert^2\, d\omega$

Some Notes:

1. There are two similar functions used to describe the functional form sin(x)/x. One is the sinc() function, and the other is the Sa() function. We will only use the sinc() notation in class. Note the role of π in the sinc() definition:

$$sinc(x) \equiv \frac{\sin(\pi x)}{\pi x}; \qquad Sa(x) \equiv \frac{\sin(x)}{x}$$

2. The impulse function, aka delta function, is defined by the following three relationships:
   a. Singularity: $\delta(t - t_0) = 0$ for all $t \neq t_0$
   b. Unity area: $\int_{-\infty}^{\infty} \delta(t)dt = 1$
   c. Sifting property: $\int_{t_a}^{t_b} f(t)\delta(t - t_0)dt = f(t_0)$ for $t_a < t_0 < t_b$.

3. Many basic functions do not change under a reversal operation. Other change signs. Use this to help simplify your results.
   a. $\delta(t) = \delta(-t)$ (in general, $\delta(at) \Leftrightarrow \frac{1}{|a|}\delta(t)$)
   b. $rect(t) = rect(-t)$
   c. $\Lambda(t) = \Lambda(-t)$
   d. $\mathrm{sinc}(t) = \mathrm{sinc}(-t)$
   e. $\mathrm{sgn}(t) = -\mathrm{sgn}(-t)$

4. The duality property is quite useful but sometimes a bit hard to understand. Suppose a known FT pair $g(t) \Leftrightarrow z(\omega)$ is available in a table. Suppose a new time function z(t) is formed with the same shape as the spectrum z(ω) (i.e. the function z(t) in the time domain is the same as z(ω) in the frequency domain). Then the FT of z(t) will be found to be $z(t) \Leftrightarrow 2\pi g(-\omega)$, which says that the F.T. of z(t) is the same shape as g(t), with a multiplier of 2π and with –ω substituted for t.

   An example is helpful. Given the F.T. pair $\mathrm{sgn}(t) \Leftrightarrow 2/j\omega$, what is the Fourier transform of x(t)=1/t? First, modify the given pair to $j/2\,\mathrm{sgn}(t) \Leftrightarrow 1/\omega$ by multiplying both sides by j/2. Then, use the duality function to show that $1/t \Leftrightarrow 2\pi\; j/2\,\mathrm{sgn}(-\omega) = j\pi\,\mathrm{sgn}(-\omega) = -j\pi\,\mathrm{sgn}(\omega)$.