# CALCULUS

## TRIGONOMETRIC DERIVATIVES AND INTEGRALS

### TRIGONOMETRIC DERIVATIVES

| | | |
|---|---|---|
| $\frac{d}{dx}(\sin(x)) = \cos(x)\cdot x'$ | $\frac{d}{dx}(\cos(x)) = -\sin(x)\cdot x'$ | $\frac{d}{dx}(\tan(x)) = \sec^2(x)\cdot x'$ |
| $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)\cdot x'$ | $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)\cdot x'$ | $\frac{d}{dx}(\cot(x)) = -\csc^2(x)\cdot x'$ |
| $\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}\cdot x'$ | $\frac{d}{dx}(\cos^{-1}(x)) = -\frac{1}{\sqrt{1-x^2}}\cdot x'$ | $\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}\cdot x'$ |
| $\frac{d}{dx}(\csc^{-1}(x)) = -\frac{1}{x\sqrt{x^2-1}}\cdot x'$ | $\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}}\cdot x'$ | $\frac{d}{dx}(\cot^{-1}(x)) = -\frac{1}{1+x^2}\cdot x'$ |

### TRIGONOMETRIC INTEGRALS

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|---|---|
| $\int \sin(x)dx = -\cos(x) + C$ | $\int \csc(x)dx = \ln\lvert\csc(x) - \cot(x)\rvert + C$ |
| $\int \cos(x)dx = \sin(x) + C$ | $\int \sec(x)dx = \ln\lvert\sec(x) + \tan(x)\rvert + C$ |
| $\int \tan(x)dx = \ln\lvert\sec(x)\rvert + C$ | $\int \cot(x)dx = \ln\lvert\sin(x)\rvert + C$ |

| POWER REDUCTION FORMULAS | INVERSE TRIG INTEGRALS |
|---|---|
| $\int \sin^n(x) = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$ | $\int \sin^{-1}(x)dx = x\sin^{-1}(x) + \sqrt{1-x^2} + C$ |
| $\int \cos^n(x) = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)dx$ | $\int \cos^{-1}(x)dx = x\cos^{-1}(x) - \sqrt{1-x^2} + C$ |
| $\int \tan^n(x) = \frac{1}{n-1}\tan^{n-1}(x) - \int \tan^{n-2}(x)dx$ | $\int \tan^{-1}(x)dx = x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2) + C$ |
| $\int \cot^n(x) = -\frac{1}{n-1}\cot^{n-1}(x) - \int \cot^{n-2}(x)dx$ | $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \sec^n(x) = \frac{1}{n-1}\tan(x)\sec^{n-2}(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)dx$ | $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \csc^n(x) = -\frac{1}{n-1}\cot(x)\csc^{n-2}(x) + \frac{n-2}{n-1}\int \csc^{n-2}(x)dx$ | $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$ |

## STRATEGY FOR EVALUATING $\int \sin^m(x)\cos^n(x)dx$

(a) If the power $n$ of cosine is odd ($n = 2k+1$), save one cosine factor and use $\cos^2(x) = 1 - \sin^2(x)$ to express the rest of the factors in terms of sine:

$$\int \sin^m(x)\cos^n(x)dx = \int \sin^m(x)\cos^{2k+1}(x)dx = \int \sin^m(x)(\cos^2(x))^k\cos(x)dx$$
$$= \int \sin^m(x)(1-\sin^2(x))^k\cos(x)dx$$

Then solve by $u$-substitution and let $u = \sin(x)$.

(b) If the power $m$ of sine is odd ($m = 2k+1$), save one sine factor and use $\sin^2(x) = 1 - \cos^2(x)$ to express the rest of the factors in terms of cosine:

$$\int \sin^m(x)\cos^n(x)dx = \int \sin^{2k+1}(x)\cos^n(x)dx = \int (\sin^2(x))^k\cos^n(x)\sin(x)dx$$
$$= \int (1-\cos^2(x))^k\cos^n(x)\sin(x)dx$$

Then solve by $u$-substitution and let $u = \cos(x)$.

(b) If both powers $m$ and $n$ are even, use the half-angle identities:

$$\sin^2(x) = \frac{1}{2}(1-\cos(2x)) \qquad \cos^2(x) = \frac{1}{2}(1+\cos(2x))$$

## STRATEGY FOR EVALUATING $\int \tan^m(x)\sec^n(x)dx$

(a) If the power $n$ of secant is even ($n = 2k, k \geq 2$), save one $\sec^2(x)$ factor and use $\sec^2(x) = 1+\tan^2(x)$ to express the rest of the factors in terms of tangent:

$$\int \tan^m(x)\sec^n(x)dx = \int \tan^m(x)\sec^{2k}(x)dx = \int \tan^m(x)(\sec^2)^{k-1}\sec^2(x)(x)dx$$
$$= \int \tan^m(x)(1+\tan^2(x))^{k-1}\sec^2(x)(x)dx$$

Then solve by $u$-substitution and let $u = \tan(x)$.

(b) If the power $m$ of tangent is odd ($m = 2k+1$), save one $\sec(x)\tan(x)$ factor and use $\tan^2(x) = \sec^2(x) - 1$ to express the rest of the factors in terms of secant:

$$\int \tan^m(x)\sec^n(x)dx = \int \tan^{2k+1}(x)\sec^n(x)dx = \int (\tan^2(x))^k\sec^{n-1}(x)\sec(x)\tan(x)dx$$
$$= \int (\sec^2(x)-1)^k\sec^{n-1}(x)\sec(x)\tan(x)dx$$

Then solve by $u$-substitution and let $u = \sec(x)$.